Physics 1A Practice Mid II

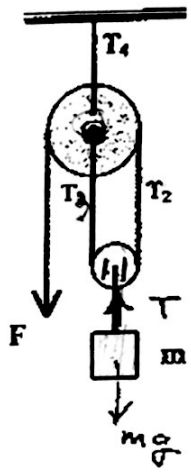
1. Mass m = 5.0 Kg is hanging from the pulley system as shown, while ropes and pulleys have negligible mass and friction. Force F is adjusted such that the block has zero acceleration.

a) Find the value of all tensions in rope sections marked and the value of force F.

$T_2 = F \quad ; \quad T_3 = T_2 \rightarrow T_2 = T_3 = F$

$mg = T_2 + T_3 = 2T_2 \rightarrow T_2 = \frac{mg}{2}$ sig figs?

$\rightarrow F = T_2 = T_3 = \frac{mg}{2} = \frac{5(9.8)}{2} = 24.5 \ N$



$T_4 = T_2 + T_3 + F$

$= T_2 + T_2 + T_2 = 3T_2$

$= 3(24.5)$ sig figs?

$\rightarrow T_4 = 73.5 \ N$

b) If the value of the force F is doubled, calculate the acceleration of mass m.

$T_2 = F \ , \ T_3 = T_2 \rightarrow F = T_2 = T_3 = 2(24.5) = 49 \ N$

in figure : $T = T_2 + T_3$

$= 2T_2 = 2(49) = 98 \ N$

$a = \frac{F_{net}}{m} = \frac{T - mg}{m} = \frac{98 - 5(9.8)}{5} = 9.8 \ m/s^2$

2. A block with mass m = 0.5 *Kg* is sliding down a frictionless track as shown. It starts from rest on top and descends down a height of h = 9.0 *m* before it goes over a half-cylinder bump with a radius of R = 7.0 *m*. Calculate:

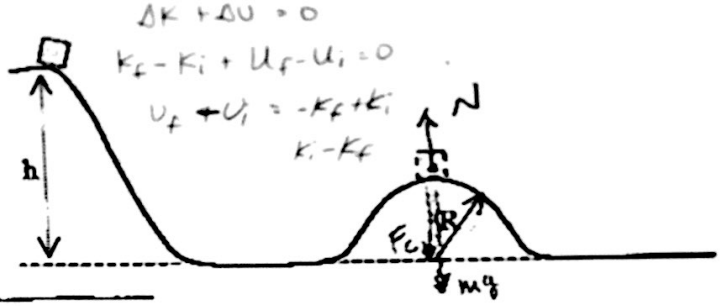
a) the velocity of the block when it arrives at the top of the half-cylinder.

$K_i + U_i = K_f + U_f$

$0 + \frac{1}{2}mgh = \frac{1}{2}mv^2 + mgR$

$v^2 = 2g(h-R)$

$v = \sqrt{2g(h-R)} = \sqrt{2(9.8)(9-7)} = 6.26 \text{ m/s}$



b) the centripetal force acting on the block when it arrives at the top of the half-cylinder.

$F_c = \frac{mv^2}{R} = \frac{0.5(6.26)^2}{7} = 2.8 \text{ N}$

c) the normal force at this point.

$F_c = mg - N \Rightarrow N = mg - F_c$

$N = (0.5)(9.8) - 2.8 = 2.1 \text{ N}$

d) What should be the minimum value for R for the block to just separate from the surface at this point.

(c) $\rightarrow \quad N = mg - F_c \leq 0$ to Separate

$$mg - m\frac{v^2}{R} \leq 0 \Rightarrow v^2 \leq Rg$$

$$\Rightarrow R \geq \frac{v^2}{g} \Rightarrow R_{min} = \frac{v^2}{g} = \frac{6.26^2}{9.8} = 4.0 \text{ m}$$

$$v^2 = 2g(h-R) \leq Rg$$

$$2h - 2R \leq R \qquad R \geq \frac{2h}{3}$$

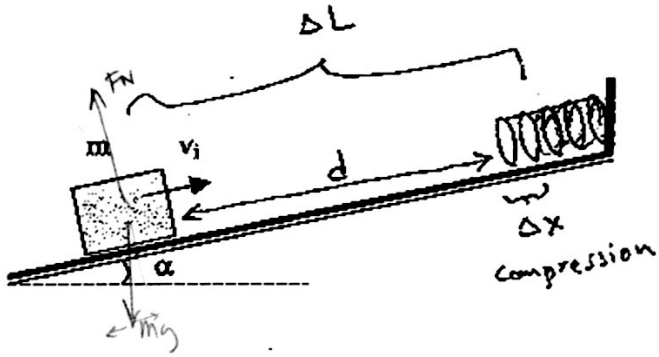
$$3R \geq 2h$$

$$R_{min} = \frac{2h}{3} = \frac{2(9.0)}{3} = 6.0 \text{ m}$$

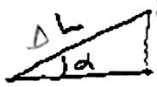
3. A block with mass m = 0.5 Kg is on a frictionless surface with a slope angle of α=15°. It is given an initial velocity of v_i = 12.0 m/s when it is a distance d = 10.0 m away from a spring with a constant k= 1200.0 N/m.

a) Find maximum amount of compression, Δx, in the spring when the mass hits it.

$$K_f + U_{g_f} + U_{s_f} = K_i + U_{g_i} + U_{s_i}$$



$$0 + mg\Delta L \sin\alpha + \frac{1}{2}k\Delta x^2 = \frac{1}{2}mv^2 + 0 + 0 \quad ; \Delta L = d + \Delta x$$



$$\rightarrow mg(d+\Delta x)\sin\alpha + \frac{1}{2}k\Delta x^2 = \frac{1}{2}mv^2$$

$$k\Delta x^2 + 2mg\sin\alpha\,\Delta x - (mv^2 - mgd\sin\alpha) = 0$$

$$\Delta x = \frac{-mg\sin\alpha \pm \sqrt{m^2g^2\sin^2\alpha + k(mv^2 - mgd\sin\alpha)}}{k}$$

$$= \frac{-(0.5)(9.8)\sin 15^\circ + \sqrt{(0.5)^2(9.8)^2\sin^2 15^\circ + 1200\left[0.5(12)^2 - (0.5)(9.8)(10)\sin\right.}}{1200}$$

$$\Delta x = 0.22 = 22 \text{ cm}$$

b) How much should the kinetic friction coefficient μ_k be, so that the mass will come to a stop just before hitting the spring?

$$W_{n.c.} = \Delta K + \Delta U = K_f - K_i + U_f - U_i$$

$$-f_r d = 0 - \frac{1}{2}mv^2 + mgd\sin\alpha - 0 \quad ; f_r = \mu_k N$$

$$-\mu_k N d = -\frac{1}{2}m(v^2 - 2gd\sin\alpha) \quad ; N = mg\cos\alpha$$

$$-\mu_k mg\cos\alpha\, d = -\frac{1}{2}m(v^2 - 2gd\sin\alpha)$$

$$\Rightarrow \mu_k = \frac{v^2 - 2gd\sin\alpha}{2g\cos\alpha\, d} = \frac{12^2 - 2(9.8)(10)\sin 15^\circ}{2(9.8)\cos 15^\circ (10)}$$

$$\Rightarrow \boxed{\mu_k = 0.49}$$

c) How much is the internal energy change in part b?

$$W_{ext} = \Delta K + \Delta E_{int}$$

$$= W_g \quad \text{because our system now includes the slope}$$

$$\Delta E_{int} = W_g - \Delta K = -\Delta U - \Delta K = -W_{n.c.}$$

$$\Delta E_{int} = f_r \cdot d = \mu_k mg\cos\alpha\, d$$

$$= 0.49(0.5)(9.8)\cos 15^\circ (10)$$

$$\rightarrow \boxed{\Delta E_{int} = 23.2\ \text{J}}$$

$\Delta E_{int} = W_{fr}$ or $-W_{fr}$

4. Small steel ball A with mass $m_A = 0.35$ Kg is attached to a massless string of length $L = 2.0$ m hanging from point P. Initially ball A is moved to the angle $\theta_A = 35°$ relative to vertical before it is given an initial velocity of $v_{iA} = 2.2$ m/s and released as shown. When it reaches the vertical position it elastically collides with the small hanging ball B with mass $m_B = 0.25$ Kg.

a) What is the velocity of ball A just as it is about to hit ball B?

$$K_f^A + U_f^A = K_i^A + U_i^A$$

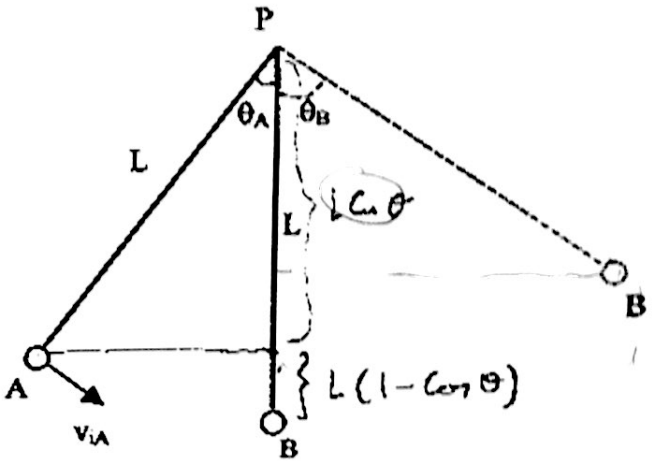
$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_{iA}^2 + mgL(1-\cos\theta_A)$$

$$v_A^2 = 2gL(1-\cos\theta_A) + v_{iA}^2$$

$$v_A = \sqrt{2gL(1-\cos\theta_A) + v_{iA}^2}$$

$$= \sqrt{2(9.8)2(1-\cos 35^\circ) + 2.2^2} = 3.45$$

$$v_A = 3.45 \text{ m/s}$$



b) Calculate the tension in the string holding ball A just as it is about to hit ball B.

$$F_c = T - mg$$

$$T = F_c + mg = m\frac{v^2}{L} + mg$$

$$= m\left(\frac{v^2}{L} + g\right)$$

$$= 0.35\left(\frac{3.45^2}{2.0} + 9.8\right) = 5.51 \text{ N}$$

3/4 sis fiss

c) Determine the maximum angle, θ_B, that ball B rises to on the opposite side.

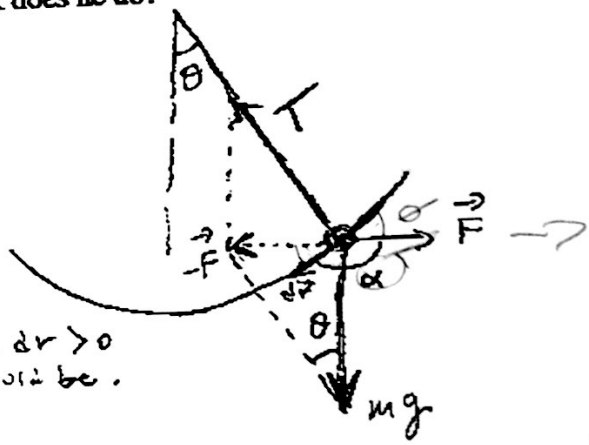
$$v'_B = \frac{2m_A}{m_A+m_B} v_A = \frac{2(0.35)}{0.35+0.25}(3.45) = 4.0 \text{ m/s}$$

$$K_f^B + U_f^B = K_i^B + U_i^B \Rightarrow \frac{1}{2}m v_B'^2 + 0 = 0 + mgL(1-\cos\theta_B)$$

$$2(gL - gL\cos\theta_B) = v_B'^2 \Rightarrow \cos\theta_B = \frac{2gL - v_B'^2}{2gL}$$

$$\Rightarrow \cos\theta_B = \frac{2(9.8)(2) - 4.0^2}{2(9.8)(2.0)} = 0.592 \Rightarrow \theta_B = 53.7$$

d) Someone captures ball B at its maximum height and gently brings it back to the vertical position by a [redacted] horizontal force. How much work does he do?



$$W = \int \vec{F}\cdot d\vec{r} \quad ; \quad F = mg\tan\theta$$

$$= \int F\cos\alpha\, dr \; ; \begin{cases} \cos\alpha = +\cos\theta \\ dr = -L\,d\theta \end{cases}$$

↑ to make $dr > 0$ as it should be.

$$W = \int_{\theta_B}^{0} (mg\tan\theta)(-\cos\theta)(-L\,d\theta)$$

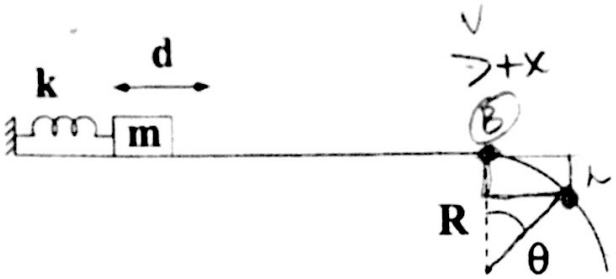
$$= mgL\int \tan\theta\cos\theta\, d\theta$$

$$= mgL\int_{\theta_B}^{0} \sin\theta\, d\theta = mgL\left[-\cos\theta\right]_{\theta_B}^{0}$$

$$= mgL\left[-1+\cos\theta_B\right]$$

$$= (0.25)(9.8)(2.0)\left[-1+\cos 53.7^\circ\right]$$

$$= -2.0 \text{ J}$$



$h = R - R\cos\theta$

1) A small block of mass m is held at rest against a spring of constant k so that it is compressed by an amount d. When released, the mass proceeds over a horizontal, frictionless plane to a frictionless, quarter-cylinder ramp of radius R.

- 1a) (10 points) Let's assume the block remains in contact with the ramp for at least a little while as it heads off the horizontal plane. Find the speed of the block as a function of its angular position on the ramp (θ, measured with respect to the vertical direction, as shown).

$$\frac{1}{2}mv_f^2 + \frac{1}{2}kx^2 + mgh = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 + mgh$$

$$v_f^2 = \frac{kd^2 + 2mg(R - R\cos\theta)}{m} = \frac{kd^2 + 2Rmg(1-\cos\theta)}{m}$$

$$v(\theta) = \sqrt{\frac{kd^2 + 2Rmg(1-\cos\theta)}{m}}$$

- 1b) (10 points) Again, assuming the block remains in contact with the curved ramp for at least a little while, find the angular position (θ) at which it would lose contact with the ramp.

$$\sum F_y = mg - F_N = ma_c$$

$$\cos\theta\, mg - F_N = m\frac{v^2}{R}$$

$$g = \frac{v^2}{R}$$

$$v = \sqrt{gR}$$

$$\sqrt{gR} = v(\theta)$$

$$gR = \frac{kd^2 + 2Rmg(1-\cos\theta)}{m}$$

$$mgR = kd^2 + 2Rmg(1-\cos\theta)$$

$$\frac{mgR - kd^2}{2mgR} - 1 = -\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-mgR + kd^2}{2mgR} + 1\right)$$

Plug into previous equation and solve for θ

- 1c) (10 points) The block actually loses contact with the ramp when $\cos(\theta) = \frac{1}{3}$. How much work was done by drag forces from the instant the block was released until the instant it left the ramp?

$\Delta E = W_{nc}$

$E_f - E_i = F_{drag}$

~~$V_f^2 = (V(\theta) \text{ in part a})^2$~~

$\frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2 - \frac{1}{2}mv_0^2 - mgh_i - \frac{1}{2}kx_i^2 = F_{drag}$

$\frac{1}{2}mV^2 + mgh_f - \frac{1}{2}kd^2 = F_{drag}$

$\frac{1}{2}m\left(\frac{kd^2 + 2Rmg\left(\frac{2}{3}\right)}{m}\right) + mgR\left(1 - \frac{1}{3}\right) - \frac{1}{2}kd^2 = F_{drag}$ $\quad V_f^2 = \frac{kd^2 + 2Rmg(1-\frac{1}{3})}{m}$

$\frac{1}{2}kd^2 + \frac{2}{3}mRg + \frac{2}{3}mgR - \frac{1}{2}kd^2 = F_{drag}$

$\frac{4}{3}mg = F_{drag}$

Wrong for part (b), but OK for (c).

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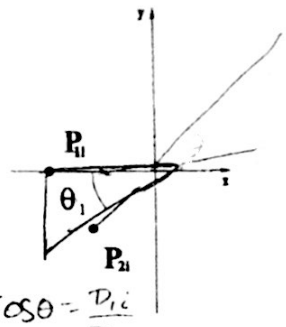
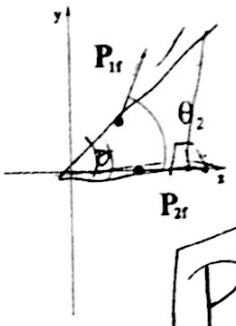


$\cos\theta = \frac{P_{ix}}{P_{i}}$



POOEY

2) Particle 1 and Particle 2 are headed for a collision at the origin. We know the magnitudes of their initial momenta ($P_{1,i}$ and $P_{2,i}$, respectively) and we know the angles separating the initial and final momenta (θ_1 and θ_2 respectively, as drawn). Further, the initial momentum of Particle 1 and the final momentum of Particle 2 both lie along the x-axis of the chosen coordinate system.

- 2a) (5 points) What physical quantities will be conserved through this collision? Why? If the definition of the system is important for any of these relationships, make sure you identify the system used.

momentum will be conserved in both x, y directions

- 2b) (10 points) Using each of these conservation relationships, write equations that connect conditions before the collision (in terms of known quantities) to conditions after the collision.

$P_{1i} + P_{2i} = P_{1f} + P_{2f}$

$P_{1ix} + P_{2ix} = P_{1fx} + P_{2fx}$

$P_{1i} + P_{2i}\left(\frac{P_{1i}}{\cos\theta_1}\right) = P_{1f}\left(\frac{P_{2f}}{\cos\theta_2}\right) + P_{2f}$

$P_{1iy} + P_{2iy} = P_{1fy} + P_{2fy}$

$0 + P_{2i}\left(\frac{P_{1i}}{\sin\theta_1}\right) = P_{1f}\left(\frac{P_{2f}}{\sin\theta_2}\right) + 0$

6

given θ_1 θ_2 P_{1i} P_{2i} SORRY

- 2c) (10 points) Find the magnitude of the final momentum for Particle 1 ($P_{1,f}$) and for Particle 2 ($P_{2,f}$).

$$P_{1i} + P_{2i}\left(\frac{P_{1i}}{\cos\theta_1}\right) = P_{1f}\left(\frac{P_{2f}}{\cos\theta_2}\right) + P_{2f}$$

$$P_{2f} = P_{1i} + P_{2i}\left(\frac{P_{1i}}{\cos\theta_1}\right) - \frac{P_{1f}P_{2f}}{\cos\theta_2}$$

$$P_{2f} = P_{1i} + P_{2i}\left(\frac{P_{1i}}{\cos\theta_1}\right) - \frac{P_{1f}\left(\frac{P_{2i}P_{1i}\sin\theta_2}{P_{2f}\sin\theta_1}\right)}{\cos\theta_2}$$

$$P_{2f} = P_{1i} + \frac{P_{2i}P_{1i}}{\cos\theta_1} - \frac{P_{1f}P_{2i}P_{1i}}{P_{2f}\sin\theta_1}\frac{\sin\theta_2}{\cos\theta_2}$$

$$P_{2f} = P_{1i} + \frac{P_{2i}P_{1i}}{\cos\theta_1} - \frac{P_{1f}P_{2i}P_{1i}}{P_{2f}\sin\theta_1}\tan\theta_2$$

$$P_{2i}\left(\frac{P_{1i}}{\sin\theta_1}\right) = P_{1f}\left(\frac{P_{2f}}{\sin\theta_2}\right)$$

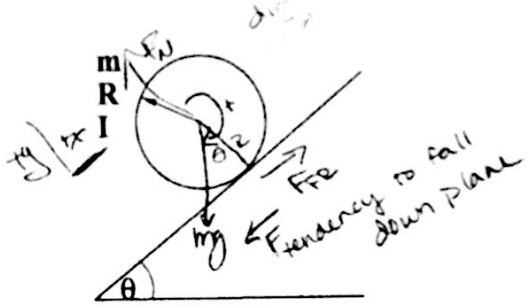
$$P_{1f} = \frac{P_{2i}P_{1i}}{\frac{\sin\theta_1}{P_{2f}}\sin\theta_2}$$

$$P_{1f} = \frac{P_{2i}P_{1i}\sin\theta_2}{P_{2f}\sin\theta_1}$$

$$P_{1f} = \frac{P_{2i}P_{1i}\sin\theta_2}{\sin\theta_1\left(P_{1i} + \frac{P_{2i}P_{1i}}{\cos\theta_1} - \frac{P_{1f}P_{2i}P_{1i}}{P_{2f}\sin\theta_1}\tan\theta_2\right)}$$

- 2d) (5 points) Evaluate $P_{1,f}$ and $P_{2,f}$ in the special case where $\theta_1 = \theta_2$. Explain your results.

When $\theta_1 = \theta_2$ the collision is elastic and KE is trans conserved as well



3) A wheel of mass m, radius R and rotational inertia I (about the longitudinal center-of-mass axis) rolls freely, without slipping, up an incline of angle θ.

- 3a) (15 points) Find the magnitude and direction of i) the center-of-mass acceleration and ii) the force of friction that acts on the wheel.

$a = R\alpha$

$\alpha = \frac{a}{R}$

$\Sigma F_y = mg\cos\theta - mg\cos\theta = ma$

$\Sigma F_x = F_{FR} - mg\sin\theta = ma$

$-F_{FR} = Ia$

$\Sigma\tau = -RF_{FR} = I\alpha$

$-RF_{FR} = Ia$

bad algebra!

$0 - mg\sin\theta = a(m+I)$

i) $a = \frac{-mg\sin\theta}{m+I}$ ($g > 0$)

$a_{cm} = \frac{mg\sin\theta}{m+I}$

ii) $-F_{FR} = Ia$

$F_{FR} = -Ia$

$= +I\left(\frac{mg\sin\theta}{m+I}\right)$

tell me direction!

$F_{FR} = -I\left(\frac{mg\sin\theta}{m+I}\right)$

- 3b) (10 points) What is the largest value of θ for which the wheel can roll up the hill without slipping?

X W/o slipping means friction must be present

$F_{FR} = 0$ → $I\left(\frac{mg\sin\theta}{m+I}\right)$ $mg\sin\theta = 0$ X

$\sin\theta = 0$ X

$[\theta = \pi]$ X

$F_{FR} = \mu_s F_n = \mu_s mg\cos\theta = I\frac{mg\sin\theta}{m+I}$

$\tan\theta = \frac{\mu_s(m+I)}{I}$

$\theta = \arctan\left(\frac{\mu_s(m+I)}{I}\right)$ ✓

which answer? II

but r doesn't make sense urgh

- 3c) (5 points) Assuming identical masses, radii, and initial speeds. as well as uniform mass distributions, which would roll farther up the hill: a hoop, a thin disk, a long cylinder or a sphere? Explain.

1 2/5 greater a slows it down!

Rolling farther requires greater acceleration

So the lowest I value would result in the fastest height. which would be a sphere

X this is why direction on a matter in part a)

(BLEH)

(1) (20 points-5 points each question)
A small 0.200 kg ball falls vertically on the floor and bounces back vertically up. Just before the impact the ball is traveling at 3.00 m/s and it leaves the ground with a speed of 2.00m/s.

1-a

what is the impulse (magnitude and direction) that acted on the ball during the collision?

1-b

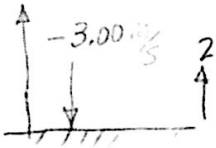
If the ball remains in contact with the floor for 0.2 s, find the average force (magnitude and direction) that acted on the ball during the collision.

1-c

What is the total work that was done on the ball during the collision?

1-d

What is the average power with which the work on the ball was done?

a) 

$I_y = \Delta p_y = 0.200\ kg \left(2.00 \frac{m}{s} + 3.00 \frac{m}{s}\right) =$

$= 1.00\ kg \frac{m}{s}$ (up-wards)

b) $\overline{F_y} = \frac{I_y}{\Delta t} = \frac{1.00\ kg \frac{m}{s}}{0.2\ s} = 5.00 N$ (up-wards)

c) $W = \Delta E_k = \frac{0.200}{2} kg \left(4.00 \frac{m^2}{s^2} - 9.00 \frac{m^2}{s^2}\right) = -0.5\ J$

d) $\overline{P} = \frac{W}{\Delta t} = \frac{-0.5\ J}{0.2\ s} = -2.5\ W$

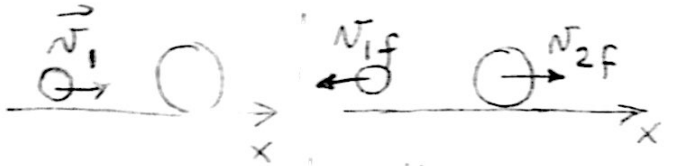
(2) (20 points-10 points each question)
A ball of mass 100.00 g and speed 10.00 m/s collides elastically with another ball of mass 1.00 kg at rest

2-a

what are the final velocities of both balls after the collision?

2-b

what is the change in momentum of each ball and the change in kinetic energy of each ball?



before ; after

a) $\vec{p}$ conservation:

$m_1 v_{1x} = m_1 v_{1fx} + m_2 v_{2fx}$ (I)

E_k conservation: $\left(\frac{m_1}{2} v_{1x}^2 = \frac{m_1}{2} v_{1fx}^2 + \frac{m_2}{2} v_{2fx}^2\right)$ or

(what we proved in class (not relative vel.)) $v_{1x} = v_{2fx} - v_{1fx}$ (II)

(I) $+ m_1$ (II) : $2 m_1 v_{1x} = (m_1 + m_2) v_{2fx} \Rightarrow$

$$v_{2fx} = \frac{2 m_1 v_{1x}}{m_1 + m_2} = \frac{2 \times 0.100\ kg}{1.10\ kg}\ 10.00 \frac{m}{s} = 1.818\ m/s$$

(I) $- m_2$ (II) : $(m_1 - m_2) v_{1x} = (m_1 + m_2) v_{1fx} \Rightarrow$

$$v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1x} = \left(\frac{-0.90}{1.10}\right) 10.00 \frac{m}{s} = -8.18\ m/s$$

b) $\Delta p_{1x} = 0.10\ kg\ (-8.18 - 10.00) \frac{m}{s} = -1.818\ kg \frac{m}{s}$

$\Delta p_{2x} = 1.00\ kg\ 1.818 \frac{m}{s} = 1.818\ kg \frac{m}{s}$

} total $\Delta p = 0$

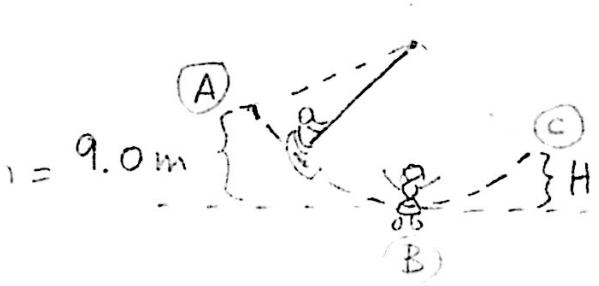
$\Delta E_{k_1} = \frac{0.10\ kg}{2} (66.9 - 100.00) \frac{m}{s} = -1.66\ J$

$\Delta E_{k_2} = \frac{1.08\ kg}{2} = 1.66\ J$

} total $\Delta E_k = 0$

(3) (10 points)

A movie stuntman (mass 80.0 kg) stands on a window ledge 9.0 m above the floor. Grabbing a rope attached to a chandelier, he swings down to save a damsel in distress (mass 40.0 kg) who is standing directly under the chandelier. He grabs the woman and both together continue the swing, until they reach another window ledge, conveniently located at the maximum hight H they can reach. (Assume that the stuntman's center of mass moves downward 9.0 m before grabbing the woman). What is the hight H?



$h = 9.0\,m$

(I) $m_1 g h = \frac{m_1}{2} v_1^2$

(II) in B: $m_1 v_1 = (m_1 + m_2) v_2$

(III) $\frac{(m_1+m_2)}{2} v_2^2 = (m_1+m_2) g H$

$$H = \frac{v_2^2}{2g} = \frac{m_1^2 v_1^2}{2g(m_1+m_2)^2} \overset{I}{=} \frac{m_1^2 \cancel{2gh}}{\cancel{2g}(m_1+m_2)^2} = \left(\frac{20.0}{50.0}\right)^2 9.00\,m$$

$$= \left(\frac{2}{3}\right)^2 9.00 = \underline{4.00\,m}$$

(4) A problem (30 points total-10 each question)

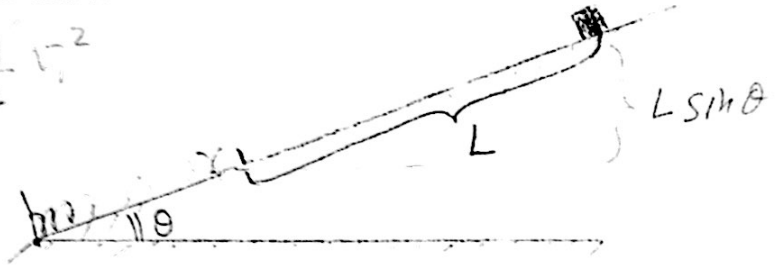
A block of mass M is given a brief push so it starts moving down an inclined plane with a velocity v_0. The plane is inclined at an angle θ.

4-a

Neglecting friction, what is the velocity v_1 of the block a distance L down the plane? Give your answer in terms of v_0, L and θ.

$$Mg\sin\theta + \frac{M}{2}v_0^2 = \frac{M}{2}v_1^2$$

$$\boxed{v_1 = (2gL\sin\theta + v_0^2)^{1/2}}$$

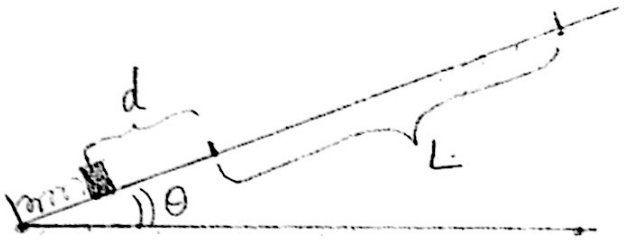


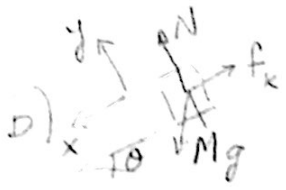
4-b

Assume now that there is friction and the kinetic friction coefficient between the block and the plane is μ_k . What is in this case the velocity v_1 a distance L down the plane? Give your answer in terms of v_0, L, θ and μ_k

4-c

Assume now that a distance L down the plane (with respect to the initial position where it had velocity v_0) the body reaches the upper end of an uncompressed spring which is parallel to the plane. The body compresses the spring by a maximum distance d. There is friction with the plane even while the body is in contact with the spring. Find the spring constant k of the spring in terms of v_0, L, θ, μ_k and d.





b) $\sum F_y$: $N = Mg\cos\theta \Rightarrow f_k = \mu_k Mg\cos\theta$

$$W_{friction} = -\mu_s Mg\cos\theta\, L = \frac{M}{2}v_1^2 - \frac{M}{2}v_0^2 - MgL\sin\theta$$

$$\Rightarrow \boxed{v_1 = (2gL(\sin\theta - \mu_s\cos\theta) + v_0^2)^{1/2}}$$

c) $$W_{friction} = -\mu_s Mg\cos\theta\,(L+d) = \frac{kd^2}{2} - \frac{M}{2}v_0^2 - Mg(L+d)\sin\theta$$

$$\boxed{k = \frac{2Mg(L+d)(\sin\theta - \mu_s\cos\theta) + Mv_0^2}{d^2}}$$

(1.) **(20 points-5 each question)**

Two friends are standing on a frictionless ice surface initially at rest. The center of mass of both friends is at a particular point between both. The friends then push each other and at a particular instant after doing so one of them (with a mass of 60kg) is at 3 m from the initial position of the center of mass. The second friend has a mass of 40 kg.

1-a

At what distance from the initial position of the center of mass is the second friend ?

1-b

If the 60 kg friend is moving at a speed of 2.0 m/s, what is the speed of the second friend at the same instant?

1-c

If the push lasted 0.2 s, what was the magnitude of the average force with which the friends pushed each other?

1-d

what was the average power delivered on each friend during the push?

a) $X_{CM} = 0 = 60\,kg \cdot 3\,m + 40\,kg\, X_2 \Rightarrow X_2 = -\frac{0.6}{0.4} 3\,m = -\frac{9}{2} m = -4.5\,m$

answer: 4.5 m

$0 = 60\,kg\, 2\,m/s + 40\,kg\, v_2 \Rightarrow v_2 = -\frac{6}{4} 2\frac{m}{s} = -3\frac{m}{s}$

answer: 3 m/s

c) $I_x = \Delta p_x = 60\,kg\, 2\frac{m}{s} - 0 = 120\,kg\frac{m}{s} = (F_1)_x \times \Delta t$

$(F_1)_x = \frac{\Delta p_x}{\Delta t} = \frac{120\,kg\frac{m}{s^2}}{0.2} = 600\,N, \quad \Delta t = 0.2\,s$

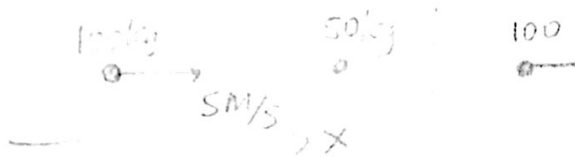
$(F_2)_x = \frac{\Delta p_x}{\Delta t} = \frac{-40\,kg \times 3\,m/s - 0}{0.2\,s} = -600\,N$

d) $W_1 = \Delta E_{K1} = 60kg\, 4\,m^2/s^2 = 120\,J, \quad P_1 = \frac{W_1}{\Delta t} = 600\,W$

$W_2 = \Delta E_{K2} = \frac{40kg}{2} 9\,m^2/s^2 = 180\,J, \quad P_2 = \frac{W_2}{\Delta t} = 900\,W$

(2) **(20 points-10 each question)**
A 100 kg man moving at 5.0 m/s collides while skating with a 50 kg woman standing on the same frictionless ice pond of the previous problem. If after the collision the man is moving in the same direction as before the collision but at 2 m/s
2-a
2,0
what is the velocity of the woman just after the collision?
2-b
what is the change in the kinetic energy of the system man plus woman?

a)



$$m_1 v_{1i_x} + m_2 v_{2i_x} = m_1 v_{1f_x} + m_2 v_{2f_x}$$

$$100\,kg\ 5\frac{m}{s} = 100\,kg\ 2\frac{m}{s} + 50\,kg\ v_{2f_x}$$

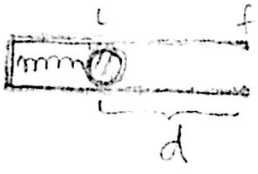
$$\Rightarrow \quad (5-2)\,kg\,\frac{m}{s} = \frac{1}{2}\,kg\,v_{2f_x} \Rightarrow \boxed{v_{2f_x} = 6\,\frac{m}{s}}$$

b) $$\Delta E_k = \frac{100}{2}\,kg\ 4\,\frac{m^2}{s^2} + \frac{50}{2}\,kg\ 36\,\frac{m^2}{s^2} - \frac{100}{2}\,kg\ 25\,\frac{m^2}{s^2}$$

$$= [200 + 900 - 1250]\,J = \boxed{-150\,J}$$

(3) (10 points)

A spring-loaded toy gun fixed to a table is used to shoot a ball of mass m horizontally. The ball is initially at rest and the spring (which has spring constant k) is compressed by a distance d. Assuming that there is a kinetic friction force f inside the gun acting on the ball during all the distance d, give the speed with which the ball leaves the gun when the ball is shot, as function of m, k, f and d.



$$W_{\text{other forces}} = E_{k_f} + U_{el_f} - (E_{k_i} + U_{el_i})$$

$$-fd = \frac{m}{2} v_f^2 + 0 - \left(0 + \frac{kd^2}{2}\right)$$

$$\Rightarrow \boxed{v_f = \sqrt{\frac{2}{m}\left(\frac{kd^2}{2} - fd\right)}}$$

$$\frac{kd^2}{m} - \frac{2fd}{m} = \frac{kd^2 - 2fd}{m}$$

(4) A problem (30 points total-10 each question)

A body of mass m is let go from a height h with respect to the table in a loop-the-loop apparatus such as the one we used in class. After falling in its track to the level of the table, the body goes up the circular part of the track, which has a radius R. The hight h is large enough so that the body maintains contact with the track all along the circular path. Assume the track is frictionless.

4-a

what is the speed of the body at the lowest point of the track?

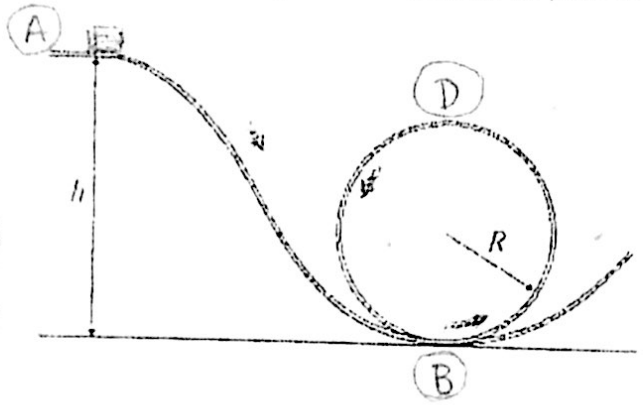
4-b

what is the minimum hight h_{min} for which the body will maintain contact with the track all along the circular path?

4-c

Assume that the body that is initially let go from the hight h_{min} collides elastically with another of identical mass which is at rest at the lowest point of the track (i.e. just where the circular part of the track begins). After the collision, will any of the two bodies complete the loop while maintaining contact all along the circular path?

Show all the equations necessary to prove each of your answers.



a) No friction $\Rightarrow$ E_{mec} = const.

$mgh = \frac{m}{2} v_B^2$, $v_B = \sqrt{2gh}$

A B

b) in (D): $a_c = \frac{v_{D\,min}^2}{r} = g$

$mgh_{min} = \frac{m}{2} v_{D\,min}^2 + mg\,2R$

A

$= \frac{m}{2} gR + mg\,2R$

$\Rightarrow h_{min} = \frac{5}{2} R$

c) at B:

v_B

① → ○

m m

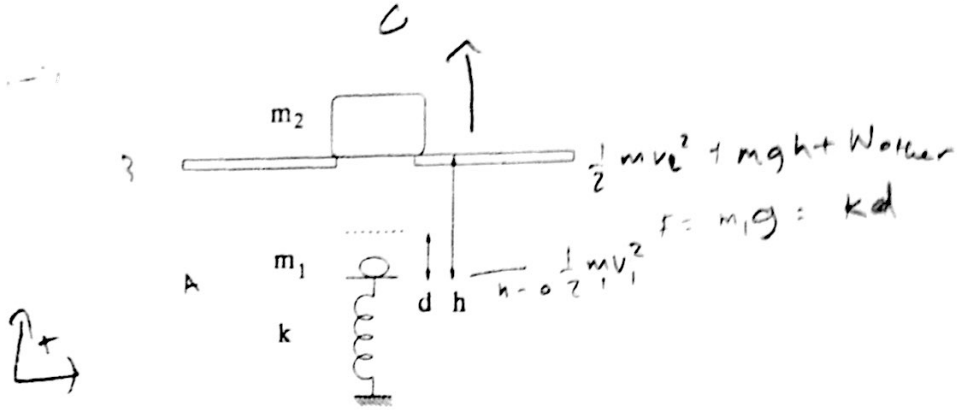
① ②

$\vec{p}$ conservation: $mv_B = mv_{1f} + mv_{2f}$ (1)

$\vec{E}_k$ conservation: $v_B = v_{2f} - v_{1f}$

(1)-(2) $\Rightarrow 0 = 2v_{1f} \Rightarrow \boxed{v_{1f} = 0}$, $\boxed{v_{2f} = v_B}$

The body 2 continues the same v in that point and it will complete the loop, so it does. (The body 1 stops dead at the point of collision and gives all its momentum to the body 2!)



1) A small piece of putty (m_1) sits in a teflon pan on a vertical spring of force-constant k that is initially held compressed by an amount d. Directly above the putty is a hole in a table, over which sits a block of mass m_2. While the spring is held compressed, the putty is located a distance h below the block.

- 1a) (10 points) The spring (and putty) are released. How fast is the putty moving when it reaches the block?

$$\Delta E = W_{nc} = 0$$

$$E_i = E_f$$

$$\frac{1}{2}kd^2 = mgh + \frac{1}{2}mV^2$$

$$kd^2 = 2m_1gh + m_1V^2$$

$$\sqrt{\frac{kd^2 - 2m_1gh}{m_1}} = V_1$$

- 1b) (10 points) If the putty sticks to the block, how far above the table will the combination rise?

(A-B)

$$\Sigma P_i = \Sigma P_f$$

$$m_1V_1 + m_2V_2 = (m_1 + m_2)V_f$$

$$\frac{m_1}{m_1 + m_2}\sqrt{\frac{kd^2 - 2m_1gh}{m_1}} = V_f$$

(B-C) $V_f = V_i$ now $V_f = 0$ when at max height

$$V_f^2 = V_i^2 + 2a\Delta y$$

$$+V_1^2 = +2g\Delta y$$

$$\frac{1}{2g}\left[\left(\frac{m_1}{m_1 + m_2}\right)^2 \frac{kd^2 - 2m_1gh}{m_1}\right] = \Delta y$$

$$\therefore \Delta y = \left(\frac{m_1}{m_1 + m_2}\right)^2 \frac{kd^2 - 2m_1gh}{2m_1g}$$

- 1c) (10 points) How much mechanical energy is lost when the putty sticks to the block? Where did it go?

deformation of the Block when the putty sticks to the Block OK

$W_i = W_f$

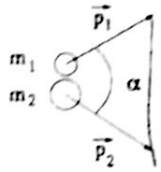
$\frac{1}{2} m_1 V_1^2 = \frac{1}{2}(m_1+m_2)V^2 + 2(m_1+m_2)h + W_{other}$

$\frac{1}{2} m_1 V_1^2 = \frac{1}{2}(m_1+m_2)\frac{(m_1)^2}{(m_1+m_2)^2}\left[\frac{kd^2-2mgh}{m_1}\right] + 2(m_1+m_2)h + W_{other}$ NO

$W_{other} = \frac{1}{2} m_1 \frac{kd^2-2mgh}{m_1} - \frac{1}{2}\frac{m_1(kd^2-2mg)}{(m_1+m_2)} - 2(m_1+m_2)h$

$W_{other} = \frac{1}{2}(kd^2-2mgh)\left[1-\frac{m_1}{m_1+m_2}\right] - 2(m_1+m_2)h$ NO

$-\ 3$



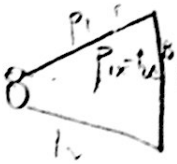
2) Two elementary particles collide elastically in a frame in which the particle of mass m_2 is initially at rest and the particle of mass m_1 is moving with a momentum $\vec{p_0}$. After the collision, m_1 has a momentum $\vec{p_1}$, m_2 has a momentum $\vec{p_2}$, and the two final state momenta are separated by an angle α (as shown).

- 2a) (10 points) Write the relevant conservation relationship(s) in terms of p_0, p_1, p_2 and α. Hint: A graphical representation may help.

$\Sigma P_{x_i} = \Sigma P_{xf}$ $\quad C^2 = A^2 + B^2 - 2AB\cos(\gamma)$ $\quad \Sigma P_{y_i} = \Sigma P_{yf}$

$P_0 = m_1 V_0$ $\quad \Sigma \vec{P}_i = \Sigma \vec{P}_f$ $\quad$ momentum is conserved

$m_1 V_0 =$



$\Sigma P_{oy} = \Sigma P_{yf} = \sqrt{P_1^2 + P_2^2 - 2P_1P_2\cos\alpha}$

$P_{xf} = 2P_{1x}$

$P_{ox} = m_1 V_{ox} = 2P_{1x}$ 5

$0 = \sqrt{P_1^2 + P_2^2 - 2P_1P_2\cos\alpha}$

$+P_1^2 + P_2^2 = 2P_1P_2\cos\alpha$

P_1 $\quad m_1 V_{ox} = 0$

$P_0^2 = P_1^2 + P_2^2 + P_1P_2\cos\alpha$ $\quad = P_1^2 + P_2^2 - 2P_1P_2\cos$

- 2b) (15 points) If $|p_1| = F|p_0|$, where $0 < F < 1$, find $\cos(\alpha)$.

$P_1^2 = P_0^2 - P_2^2 - P_1P_2\cos\alpha$ 4

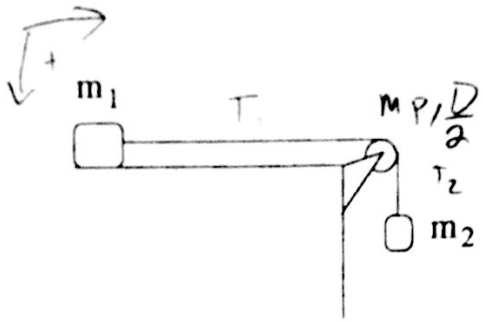
$|P_1| = \sqrt{P_0^2 - P_2^2 - P_1P_2\cos\alpha} = F|P_0|$

$\frac{P_0^2}{2m_1}$

- 2b) (Continued)

- 2c) (5 points) Check your answer to part b in the limit $m_1 = m_2$. In billiards, the cue ball has a slightly different mass than the other balls. Is it possible to predict the scattering angle, α, ahead of time? Explain.

$m_1 = m_2$ $\lim \to 0$ then the first ball stops, and the angle is predictable. can approach an angle of 90°



3) (**Y&F 10.16**) A box of mass m_1 resting on a horizontal, frictionless surface is attached to a weight m_2 by a thin, light wire that passes over a frictionless pulley. The pulley has the shape of a uniform solid disk of mass m_p and diameter D. After the system is released, find

- 3a) (10 points) the tension in the wire on both sides of the pulley,

$\sum F_{2y} = m_2 a$ $\quad \sum F_{1x} = m_1 a$ $\quad R = \frac{D}{2}$

$m_2 g - T_2 = m_2 a$ $\quad T_1 = m_1 a$

$a = r\alpha$ $\quad \frac{a}{r}$

$\sum \tau = I\alpha$

$(T_2 - T_1)R = \frac{1}{2} M_p R^2 \alpha$

$T_2 - T_1 = \frac{1}{2} M_p \frac{a}{R}$

$T_2 - T_1 = \frac{1}{2} M_p$

α = without slipping where $\alpha = \frac{a}{r}$

$m_2 g - T_2 = m_1 a$

$T_1 = m_1 a$

$T_2 - T_1 = \frac{1}{2} M_p a$ -3

$m_2 g = \left(m_1 + m_2 + \frac{1}{2} M_p\right) a$

$a = \frac{m_2 g}{\left(m_1 + m_2 + \frac{1}{2} M_p\right)}$

$T_1 = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2 + \frac{1}{2} M_p}$

$T_2 = m_2 g - m_2 \left(\frac{m_2 g}{m_1 + m_2 + \frac{1}{2} M_p}\right)$

- 3b) (10 points) the acceleration of the box, and

$a = \frac{m_2 g}{m_1 + m_2 + \frac{1}{2} m_p}$ refer above.

- 3c) (10 points) the horizontal and vertical components of the force that the axle exerts on the pulley.

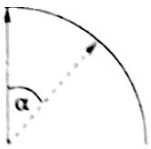
$$\Sigma F_y = m_2 a$$

$$F_y = \frac{m_2^2 g}{\left(m_1 + m_2 + \frac{1}{2} M_p\right)}$$

$$\Sigma F_x = m_1 g$$

$$= \frac{m_1 m_2}{\left(m_1 + m_2 + \frac{1}{2} m_p\right)}$$

-10



(7)

1) A pair of masses, m_1 and m_2, are held in position on either side of a compressed spring by a light string. When the string is cut, m_1 slides a distance d over a patch of friction (μ_k) before coming to rest.

- 1a) (10 points) How much energy was stored in the spring before the string was cut?

$U_{sp} = \frac{1}{2}kd^2$

$\sum F_x = m_1 a_x$

$f_s = m_1 a_x$

Spring energy to kinetic

$\Delta E = W_{NC}$

$W_{NC} = \int F \cdot d\vec{r}$

$= \int |F||dr|\cos(\theta)$

$K_{f_{m_1}} + U_{e_{sp}} - K_{i_{m_1}} - U_{i_{sp}} = -\frac{1}{2}kd^3$

$\theta = 180$

$\cos = -1$

m_1 starts and ends at rest

$-\frac{1}{2}kd^2 = -\frac{1}{2}kd^3$

$= -\frac{1}{2}kd^2 \int dr$

$= -\frac{1}{2}kd^3$

Energy stored = E^*

- 1b) (10 points) At what value of α will m_2 lose contact with the frictionless cylindrical ramp of radius R?

1

If loss of contact

$a_{tan} > a_{centri}$

$> \frac{v^2}{R}$

- 1c) (10 points) Suppose m_1 and m_2 are *both* subject to drag (air resistance). How will this affect the angle at which m_2 loses contact with the cylindrical ramp? (Hint: You do not have to calculate the actual work done by drag on either object, you do have to account for the work done by drag).

If m_2 is subject to drag, it will remain on the ramp for a larger angle because it must overcome more force to leave it. }

2) In class, we discussed what happens when two particles of identical mass undergo a glancing elastic collision. Now let's consider what happens when the two particles have different mass. For the following assume the incident particle has mass m_1, the target particle has a mass m_2, **and** the incident particle transfers to the target particle a fraction $\mathcal{F}$ of its original kinetic energy.

- 2a) (10 points) Find the ratio of the final momenta, $\frac{p_3}{p_1}$.

$c^2 = A^2 + B^2 - 2AB\cos(C)$ $\cos(C) = \cos(180 - \alpha)$

$c^2 = A^2 + B^2 + 2AB\cos(\alpha)$

$p_3^2 = p_1^2 + p_2^2 + 2p_1p_2\cos(\alpha)$

$\frac{p_3^2}{p_1^2} = p_1^2 + p_2^2 + 2p_1p_2\cos(\alpha)$

momentum conserved

$p_3 = p_1 + p_2$

$p_3^2 = p_1^2 + 2p_1p_2 + p_2^2$

(10)

Law of sines

$\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)}$

elastic

$K_f = K_i$

$\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2$

$m_1v_{1f}^2 + m_2v_{2f}^2 = m_1v_{1i}^2$

- 2b) (10 points) Find the scattering angle, α between the final momenta. Check that it gives the correct value for the case $m_1 = m_2$.

$m_2v_{2f}^2 = \mathcal{F}m_1v_{1i}^2$

$m_1v_{1f}^2 = m_1v_{1i}^2 - \mathcal{F}m_1v_{1i}^2$

$v_{1f}^2 = v_{1i}^2(1-\mathcal{F})$

$v_{1f} = v_{1i}\sqrt{1-\mathcal{F}}$

$m_1v_{1f} = m_1v_{1i}\sqrt{1-\mathcal{F}}$

$p_1 = p_3\sqrt{1-\mathcal{F}}$

$\frac{p_1}{\sqrt{1-\mathcal{F}}} = p_3$

If $m_1 = m_2$ and momentum is conserved,

$K_f = K_i$

$\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_1v_{2i}^2$

becomes

$mv_{1f}^2 + mv_{2f}^2 = mv_{1i}^2$

$m^2v_{1f}^2 + m^2v_{2f}^2 = m^2v_{1i}^2$

$p_1^2 + p_2^2 = p_3^2$

$p_3^2 = p_1^2 + p_2^2 + 2p_1p_2\cos(\alpha)$

$p_3^2 = p_1^2 + p_2^2 + 2p_1p_2\cos(\alpha)$

$0 = 2p_1p_2\cos(\alpha)$

$0 = \cos(\alpha)$

90° ($\pi/2$) when $m_1 = m_2$

(6)

- 2c) (5 points) Examine your answer to part b in the limit $\mathfrak{F} \to 1$. What is happening here? Explain.

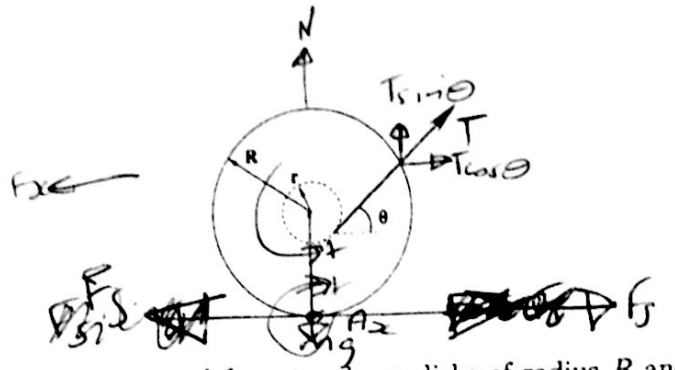
As F→, particle 1 transfers more and more of its kinetic energy to particle 2. If F = 1, particle 1 would come to rest and particle 2 would slide away with all the energy.

(2)

- 2d) (5 points) Given that the cue ball (m_1) is a little heavier than the other billiard balls (m_2), what does your answer to part b suggest with regard to being able to predict the direction of the target ball after a collision with the cue ball?

You cannot certainly predict the direction of the target ball after the collision because energy is lost to sound during the collision and different felts on pool tables also affect it.

(1)



3) A yo-yo of mass m is constructed from two large disks of radius R and a small axle of radius r. The rotational inertia about its symmetry axis is given by I_{cm}. A light string, wound around the axle as shown, is given a small tension T in a direction θ above the horizontal. Assume there is more than enough friction to prevent the yo-yo from slipping on the ground when/if it rolls.

- 3a) (10 points) What is the magnitude of the yo-yo's center-of-mass acceleration? In what direction will it accelerate?

$r_\perp T = \tau_T$

Torque at pt. of rot $\Rightarrow r_\perp = 0$ so $\tau = 0$

$\Sigma\tau = I\alpha$

$rT = \frac{1}{2}mR^2 \cdot \frac{a_x}{R}$

$rT = \frac{1}{2}mRa_x$

$\frac{2rT}{mR} = a_x$

It will accelerate to the right to counteract the axle's rotation

- 3b) (10 points) What is the magnitude of the force of friction acting on the yo-yo? In what direction does it point?

$f_s \leq \mu_s N$

Friction is pointing to the right so that it can oppose the forward motion of the yo-yo.

- 3c) (10 points) Suppose the string is pulled vertically. What is the largest horizontal acceleration the yo-yo may achieve?